Unit –II: Thermal Physics

1.Introduction
1.1Modes of Heat Transfer

Normally there are three modes of transfer of heat from one place to another viz., conduction, convection and radiation.

Conduction: Conduction refers to the heat transfer that occurs across the medium. Medium can be solid or a fluid.

Convection: It is the process in which heat is transferred from hotter end to colder end by the actual movement of heated particles.

.**Radiation:** In radiation, in the absence of intervening medium, there is net heat transfer between two surfaces at different temperatures in the form of electromagnetic waves.

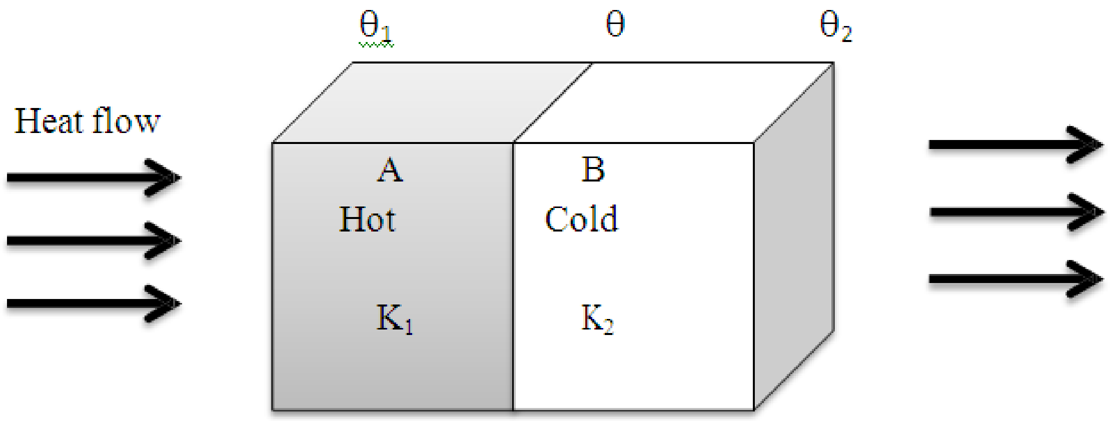
1.2 Specific Heat Capacity

The specific heat capacity of a material is the amount of energy (in Joules) needed to increase the temperature of one kilogram of mass of the material by one Kelvin.

2. HEAT CONDUCTION THROUGH A COMPOUND MEDIA (SERIES AND PARALLEL)

2.1Series

Consider a composite slab of two different materials, A & B of thermal conductivity K_1 & K_2 respectively. Let the thickness of these two layers A & B be d_1 and d_2 respectively



Let the temperature of the end faces be θ_1 & θ_2 and temperature at the contact surface be θ, which is unknown. Heat will flow from A to B through the surface of contact only if $\theta_1 > \theta_2$. After steady state is reached heat flowing per second (Q) through every layer is same. A is the area of cross section of both layers

Amount of heat flowing per sec through A

$$Q = \frac{K_1 A(\theta_1 - \theta)}{x_1} \quad \ldots\ldots\ldots\ldots(1)$$

Amount of heat flowing per sec through B

$$Q = \frac{K_2 A(\theta_1 - \theta)}{x_1} \quad \ldots\ldots\ldots\ldots(2)$$

The amount of heat flowing through the materials A and B is equal in steady conditions

Hence (1) and (2) are equal

$$\frac{K_1 A(\theta_1 - \theta)}{x_1} = \frac{K_2 A(\theta_1 - \theta)}{x_1} \quad \ldots\ldots\ldots\ldots(3)$$

Rearranging the (3), we have

$$K_1 A(\theta_1 - \theta)x_2 = K_2 A(\theta - \theta_2)x_1$$

$$K_1\theta_1 x_2 - K_1\theta x_2 = K_2\theta x_1 - K_2\theta_2 x_1$$

$$K_1\theta_1 x_2 + K_2\theta_2 x_1 = K_2\theta x_1 + K_1\theta x_2$$

$K_1\theta_1 x_2 + K_2\theta_2 x_1 = \theta(K_2 x_1 + K_1 x_2)$

$$\theta = \frac{K_1\theta_1 x_2 + K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2} \ldots\ldots\ldots..(4)$$

This is the expression for interface temperature of two composite slabs in series.

Substituting θ from equation (4) in equation (1), we get

$$Q = \frac{K_1 A}{x_1}\left[\theta_1 - \left(\frac{K_1\theta_1 x_2 + K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$= \frac{K_1 A}{x_1}\left[\left(\frac{K_2\theta_1 x_1 + K_1\theta_1 x_2 - K_1\theta_1 x_2 - K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$= \frac{K_1 A}{x_1}\left[\frac{K_2\theta_1 x_1 - K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right]$$

$$= \frac{K_1 K_2 A}{x_1}\left[\frac{\theta_1 x_1 - \theta_2 x_1}{K_2 x_1 + K_1 x_2}\right]$$

$$= \frac{K_1 K_2 A(\theta_1 - \theta_2)}{K_2 x_1 + K_1 x_2}$$

$$= \frac{A(\theta_1 - \theta_2)}{\frac{K_2 x_1}{K_1 K_2} + \frac{K_1 x_2}{K_1 K_2}}$$

$$Q = \frac{A(\theta_1 - \theta_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}} \ldots\ldots\ldots\ldots\ldots.. (5)$$

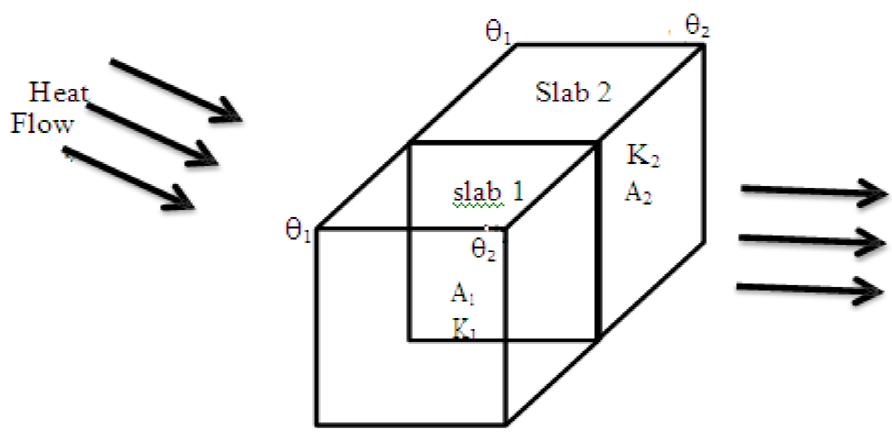
'Q' is the amount of heat flowing through the compound wall of two materials. This method can also be extended to composite slab with more than two slabs.

Generally, the amount of heat conducted per sec for any number of slabs is given by ,

$$Q = \frac{A(\theta_1 - \theta_2)}{\Sigma\left(\frac{x}{K}\right)}$$

2.2PARALLEL

Let us consider a compound wall of two different materials A and B of thermal conductivities K_1 and K_2 and of thickness d_1 and d_2 respectively. These two material layers are arranged in parallel.



The temperatures θ_1 is maintained at one faces of the material A and B and opposite faces of the material A and B are at temperature θ_2. A_1 & A_2 be the areas of cross-section of the materials.

Amount of heat flowing through the first material (A) in one second.

$$Q_1 = \frac{K_1 A_1(\theta_1 - \theta_2)}{x_1} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Amount of heat flowing through the second material (B) in one second.

$$Q_2 = \frac{K_2 A_2(\theta_1 - \theta_2)}{x_2} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

The total heat flowing through these materials per second is equal to the sum of Q_1 and Q_2

$$Q = Q_1 + Q_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substituting equations (1) and (2) in (3), we get,

$$Q = \frac{K_1 A_1 (\theta_1 - \theta_2)}{x_1} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{x_2}$$

$\therefore$ Amount of heat flowing per second

$$Q = (\theta_1 - \theta_2)\left(\frac{K_1 A_1}{x_1} + \frac{K_2 A_2}{x_2}\right)$$

In general, the net amount of heat flowing per second parallel to the composite slabs is given by

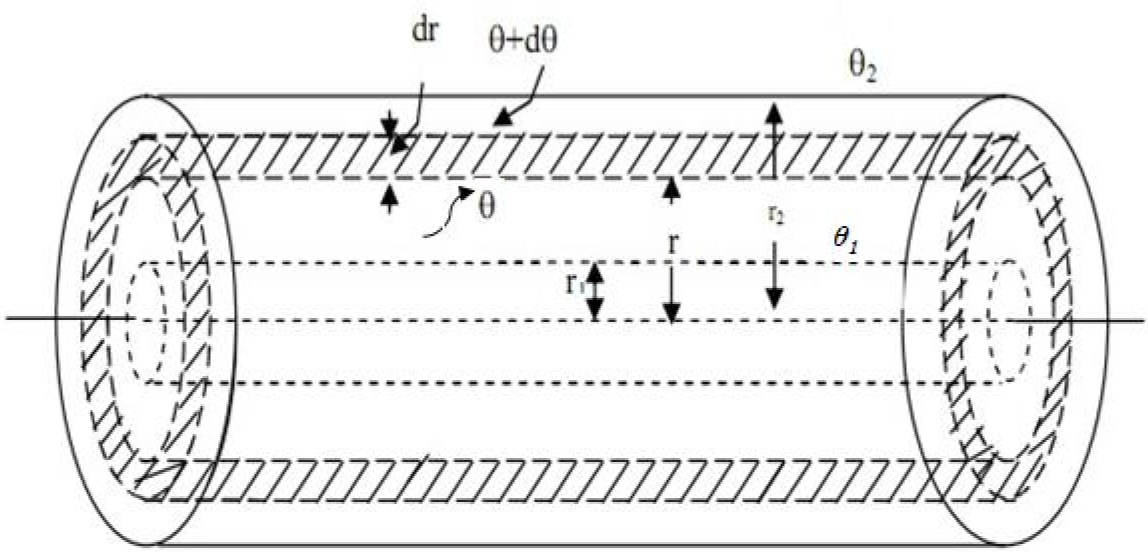
$$\boxed{\Sigma Q = (\theta_1 - \theta_2)\Sigma \frac{KA}{x}}$$

3. RADIAL FLOW OF HEAT

In this method heat flows from the inner side towards the other side along the radius of the cylindrical shell. This method is useful in determining the thermal conductivity of bad conductors taken in the powder form.

3.1 CYLINDRICAL SHELL METHOD (or) RUBBER TUBE METHOD

Consider a cylindrical tube of length l, inner radius r_1 and outer radius r_2. The tube carries steam or some hot liquid. After the steady state is reached, the temperature on the inner surface is θ_1 and on the outer surface is θ_2 in such a way $\theta_1 > \theta_2$. Heat is conducted radially across the wall of the tube. Consider an element of thickness dr and length l at a distance r from the axis.



3.2 **Working:**

Steam is allowed to pass through the axis of the cylindrical shell. The heat flows from the inner surface to the other surface radially. After the steady state is reached, the temperature at the inner surface is noted as $\theta 1$ and on the outer surface is noted as $\theta 2$.

Calculation:

The cylinder may be considered to be consisted of a large number of co-axial cylinders of increasing radii. Consider such an elemental cylindrical shell of the thickness dr at a distance 'r' from the axis. Let the temperatures of inner and outer surfaces of the elemental shell be θ and $\theta+d\theta$. Then,

The Amount of heat conducted per second $Q = -KA\frac{d\theta}{dr}$

Here Area of cross section $A = 2\pi r\, l$

$$\therefore Q = -2\pi r l K \frac{d\theta}{dr}$$

Rearranging the above equation we have

$$\frac{dr}{r} = \frac{-2\pi l K}{Q} d\theta \qquad \ldots\ldots\ldots\ldots(1)$$

∴ The Thermal conductivity of the whole cylinder can be got by, integrating equation (1) within the limits r1 to r2 and θ1 to θ2,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-2\pi l K}{Q} \int_{\theta_1}^{\theta_2} d\theta$$

$$\log_e\left(\frac{r_2}{r_1}\right) = \frac{2\pi l K}{Q}(\theta_1 - \theta_2)$$

Rearranging we get,

$$K = \frac{Q.\log_e\left(\frac{r_2}{r_1}\right)}{2\pi l(\theta_1 - \theta_2)}$$

$$K = \frac{Q \times 2.3026 \times \log_{10}\left(r_2/r_1\right)}{2\pi l\,(\theta_1 - \theta_2)} \text{ W m}^{-1}\text{K}^{-1}$$

By knowing the values in RHS, the thermal conductivity of the given material can be found.

3.3 DETERMINATION OF THERMAL CONDUCTIVITY OF RUBBER

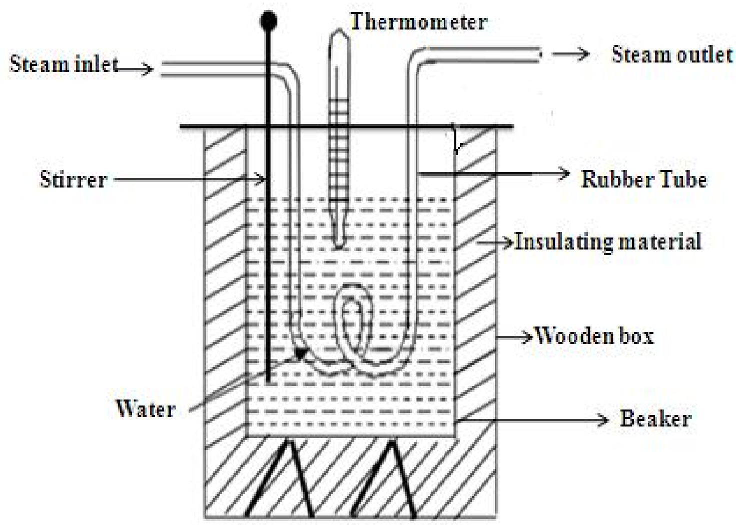
It is based on the principle of radial flow of heat through a cylindrical shell.

3.3.1Description: It consists of a calorimeter, stirrer with a thermometer. The setup is kept inside the wooden box. The space between the calorimeter and the box is filled with insulating materials such as cotton, wool, etc. to avoid radiation loss, as shown in fig.

3.3.2Working:

- The empty calorimeter is weighed, let it be (w_1).
- It is filled with two third of water and is again weighed, let it be (w_2)

- A known length of rubber tube is immersed inside the water contained in the calorimeter.
- Steam is passed through one end of the rubber tube and let out through the other end of the tube.
- The heat flows from the inner layer of the rubber tube to the outer layer and is radiated.
- The radiated heat is gained by the water in the calorimeter.
- The time taken for the steam flow to raise the temperature of the water about 10°C is noted, let it be t' seconds.



Observation and calculation:

Let $w1$	$\longrightarrow$	**Weight of calorimeter**
w_2	$\longrightarrow$	**Weight of calorimeter and water**
$w2 - w1$	$\longrightarrow$	**Weight of the water alone**
θ_1	$\longrightarrow$	**Initial temperature of the water**
θ_2	$\longrightarrow$	**Final temperature of the water**
$\theta_2 - \theta_1$	$\longrightarrow$	**Rise in temperature of the water**
θ_S	$\longrightarrow$	**Temperature of the steam**
l	$\longrightarrow$	**Length of the rubber tube (immersed)**
r_1	$\longrightarrow$	**Inner radius of the rubber tube**
r_2	$\longrightarrow$	**Outer radius of the rubber tube**
s_1	$\longrightarrow$	**Specific heat capacity of the calorimeter**
s_2	$\longrightarrow$	**Specific heat capacity of the water**

$\theta_3 \longrightarrow$ **Average temperature of the rubber tube.**

$$\theta_3 = \frac{\theta_1+\theta_2}{2}$$

We know from the theory of cylindrical shell method the amount of heat conducted by the rubber tube per second is given by

$$Q = \frac{K2\pi l(\theta_S-\theta_3)}{\log_e\left(\frac{r_2}{r_1}\right)} \text{ ....(1)}$$

The amount of heat gained by calorimeter per second $= \dfrac{w_1 s_1(\theta_2-\theta_1)}{t}$ **...............(2)**

The amount of heat gained by water per second $= \dfrac{(w_2-w_1)s_2(\theta_2-\theta_1)}{t}$ **......(3)**

$\therefore$ **The amount of heat gained by the water and calorimeter per second is obtained by (2) +(3)**

$$Q = \frac{(w_2-w_1)s_2(\theta_2-\theta_1)+w_1s_1(\theta_2-\theta_1)}{t}$$

$$Q = \frac{(\theta_2-\theta_1)[w_1s_1+(w_2-w_1)s_2]}{t} \text{ ....(4)}$$

Under steady state

The amount of heat conducted by tube per second = The amount of heat gained by the rubber the water and the calorimeter per second

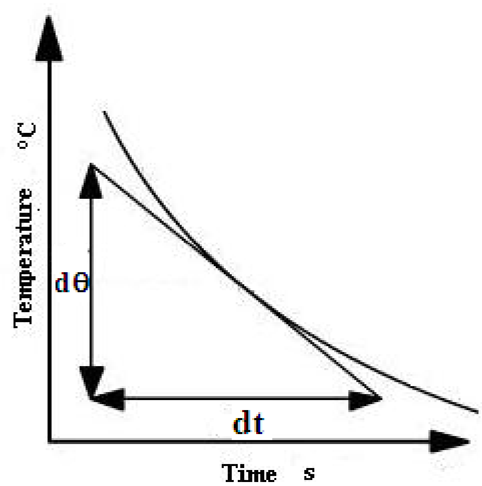
Hence, equation (1) = Equation (4)

$$\frac{K2\pi l(\theta_S-\theta_3)}{\log_e\left(\frac{r_2}{r_1}\right)} = \frac{(\theta_2-\theta_1)[w_1s_1+(w_2-w_1)s_2]}{t}$$

Substituting $\theta_3 = \dfrac{\theta_1+\theta_2}{2}$

$$\therefore \qquad K = \frac{(\theta_2-\theta_1)\log_e\left(\frac{r_2}{r_1}\right)\left[w_1 s_1+(w_2-w_1)s_2\right]}{2\pi lt\left[\theta_s-\frac{(\theta_1+\theta_2)}{2}\right]} \quad Wm^{-1}K^{-1}$$

By substituting the values in RHS, the thermal conductivity of the rubber can be determined.



3.4 Methods to determine thermal conductivity

The thermal conductivity of a material is determined by various methods

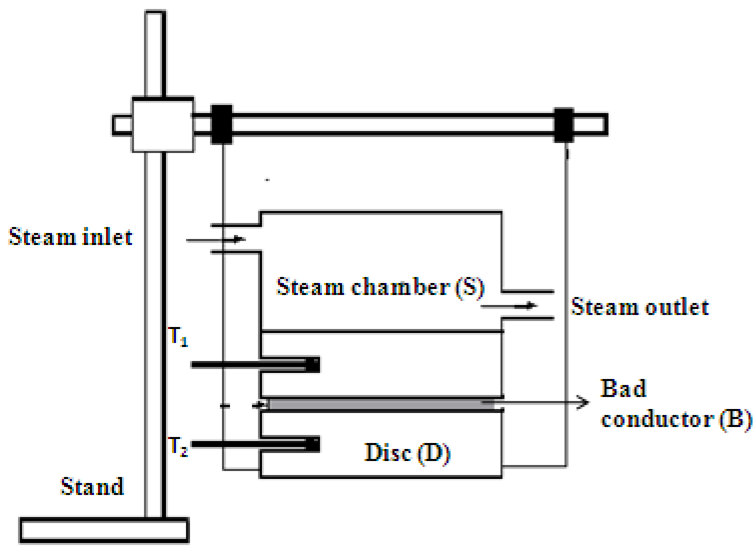
1. Searle's method – for good conductors like metallic rods
2. Forbe's method - for determining the absolute conductivity of metals
3. Lee's disc method – for bad conductors
4. Radial flow method – for bad conductors

3.5 LEE'S DISC METHOD FOR DETERMINATION OF THERMAL CONDUCTIVITY OF BAD CONDUCTOR

The thermal conductivity of bad conductor like ebonite or card board is determined by this method**.**

3.5.1 Description:

The given bad conductor (B) is shaped with the diameter as that of the circular slab (or) disc 'D'. The bad conductor is placed in between the steam chamber (S) and the disc (D), provided the bad conductor, steam chamber and the slab should be of same diameter. Holes are provided in the steam chamber (S) and the disc (D) in which thermometer are inserted to measure the temperatures. The total arrangement is hanged over the stand as shown in fig.



3.5.2 Working:

Steam is passed through the steam chamber till the steady state is reached . Let the temperature of the steam chamber (hot end) and the disc (cold end) be θ_1 and θ_2 respectively**.**

3.5.3Observation and Calculation:

Let 'x' be the thickness of the bad conductor (B), 'm' is the mass of the slab, 's' be the specific heat capacity of the slab. 'r' is the radius of the slab and 'h' be the height of the slab, then

Amount of heat conducted by the

Bad conductor per second $= \dfrac{KA(\theta_1-\theta_2)}{x}$ …….(1)

Area of the cross section is $= \pi r^2$ ……………… (2)

Amount of heat conducted per second $= \dfrac{K\pi r^2(\theta_1-\theta_2)}{x}$ ……(3)

The amount of heat lost by slab per second = m x s x Rate of cooling

$= msR_c$…………..(4)

Under steady state

The amount of heat conducted by the Bad conductyor (B) per second = Amount of heat lost by the slab (D) Per second

Hence, we can write equation (3) = equation (4)

$$\frac{K\pi r^2(\theta_1-\theta_2)}{x} = msR_c$$

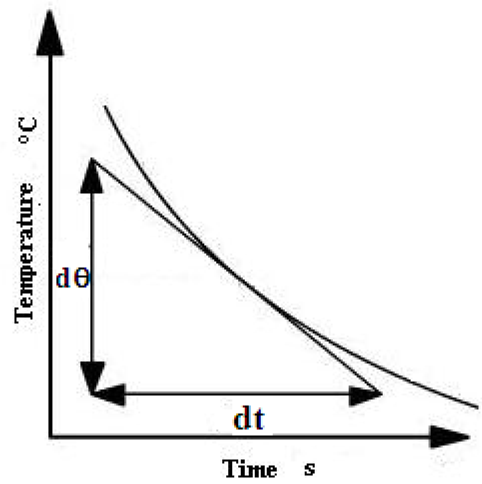
$$K = \frac{msxR_c}{\pi r^2(\theta_1-\theta_2)} \quad …….(5)$$

To find the rate of cooling R_c

R_c in equation (3) represents the rate of cooling of the disc along with the steam chamber. To find the rate of cooling for the disc alone, the bad conductor is removed and the steam chamber is directly placed over the disc and heated.

When the temperature of the slab attains 5°C higher than θ_2, the steam chamber is removed. The slab is allowed to cool, simultaneously a stop watch is switched ON.

A graph is plotted taking time along 'x' axis and temperature along 'y' axis, the rate of cooling for the disc alone (i.e) $\left(\dfrac{d\theta}{dt}\right)$ is found from the graph as shown in fig.



The rate of cooling is directly proportional to the surface area exposed.

Case(i)

Steam chamber and bad conductor are placed over slab, in which radiation takes place from the bottom surface of area (πr^2) of the slab and the sides of theof area ($2\pi rh$).

$$\therefore\ R_c = 2\pi r^2 + 2\pi rh$$

$$R_c = \pi r(r+2h)\ldots\ldots\ldots(6)$$

Case(ii)

The heat is radiated by the slab alone, (i.e) from the bottom of area(πr^2), top surface of the slab of area (πr^2) and also through the sides of the slab of area $2\pi rh$.

$$\therefore \left(\frac{d\theta}{dt}\right)_{\theta_2} = \pi r^2 + \pi r^2 + 2\pi rh$$

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2\pi r^2 + 2\pi rh$$

$$\left(\frac{d\theta}{dt}\right)_{\theta_2} = 2\pi r(r+h)\ldots\ldots..(7)$$

From (6) and (7)

$$\frac{R_c}{\left(\frac{d\theta}{dt}\right)_{\theta_2}} = \frac{\pi r(r+2h))}{2\pi r(r+h))}$$

$$R_c = \frac{(r+2h)}{2(r+h)}\left(\frac{d\theta}{dt}\right)_{\theta_2} \quad \ldots\ldots\ldots(8)$$

Substituting (8) in (5) we have

$$K = \frac{msx\left(\frac{d\theta}{dt}\right)_{\theta_2}(r+2h)}{\pi r^2(\theta_1-\theta_2)2(r+h)} \ \mathrm{Wm^{-1}K^{-1}}$$

Hence, thermal conductivity of the given bad conductor can be determined from the above relation.