Q. Code: 920315

Reg. No.

B.E. / B.TECH. DEGREE EXAMINATIONS, DEC 2019

Third Semester

MA16351 – MATHEMATICS - III

(Common to all branches)

(Regulation 2016)

Time: Three Hours **Maximum : 100 Marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

		CO	RBT
1.	Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.	**1**	**AP**
2.	Solve $(D^2 + DD' - 2D'^2)z = 0$.	**1**	**AP**
3.	State the conditions for the existence of Fourier series of a function $f(x)$.	**2**	**R**
4.	Find the coefficient b_n of the Fourier series for the function $f(x) = x\sin x$ in $(-2,2)$.	**2**	**U**
5.	Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$	**3**	**AN**
6.	Write down the three possible solutions of Laplace equation in two dimensions.	**3**	**R**
7.	State Fourier integral theorem.	**4**	**R**
8.	Find the finite Fourier sine transform of $\left(\frac{x}{\pi}\right)$ in $(0,\pi)$.	**4**	**U**
9.	State initial and final value theorem for Z-transform.	**5**	**R**
10.	Form a difference equation form $y_n = A + B3^n$ by eliminating the arbitrary constants A and B.	**5**	**U**

PART B - (5 X16 = 80 Marks)

11.	(a)	(i)	Form the partial differential equation by eliminating 'f' from $f(x^2 + y^2 + z^2,\, x + y + z) = 0$.	**(8)**	**1**	**AP**
		(ii)	Find the singular solution of $z = px + qy + p^2q^2$.	**(8)**	**1**	**AP**
			(OR)			
	(b)	(i)	Solve $(x^2 - y^2 - z^2)p + 2xyq = 2zx$.	**(8)**	**1**	**AP**
		(ii)	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial xy} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y) + e^{x-y}$.	**(8)**	**1**	**AP**
12.	(a)	(i)	Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.	**(8)**	**2**	**AP**
		(ii)	Obtain the Fourier series for $f(x) = \mathrm{x}^2$ in $-\pi < x < \pi$. Hence show that $\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots.$	**(8)**	**2**	**AP**
			(OR)			

(b) (i) Obtain a half range cosine series for $f(x)=\begin{cases} kx, & \text{in } 0\le x\le \frac{l}{2} \\ k(l-x), & \text{in } \frac{l}{2}\le x\le l \end{cases}$. **(8)** **2** **AP**

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by the following table: **(8)** **2** **AP**

x	0	π /3	2 π/ 3	π	4 π/3	5 π/ 3	2 π
f(x)	1	1.4	1.9	1.7	1.5	1.3	1

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$, find the displacement of any point on the string at a distance x from one end at any time t. **(16)** **3** **AP**

(OR)

(b) An insulated rod of length $'l'$ has its ends A and B maintained at $0^o c$ and $100^o c$ respectively until steady state conditions prevail. If B is suddenly reduced to $0^o c$ and maintained at $0^o c$, find the temperature at a distance x from A at time t. **(16)** **3** **AP**

14. (a) (i) Find Fourier transform of $e^{-\frac{x^2}{2}}$. **(10)** **4** **AP**

(ii) Find the Fourier cosine transform of $f(x)=\begin{cases} x, & for\ 0<x<1 \\ 2-x, & for\ 1<x<2. \\ 0, & for\ x>2 \end{cases}$ **(6)** **4** **AP**

(OR)

(b) (i) Find the Fourier transform of f (x) defined as **(10)** **4** **AP**

$$f(x)=\begin{cases} a-|x|, & for\ |x|<a \\ 0 & ,\ for\ |x|>a>0. \end{cases}$$

Hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt.$

(ii) Find the Fourier sine transform of $f(x)=\begin{cases} \sin x, & 0<x<a \\ 0, & \text{x > a} \end{cases}$. **(6)** **4** **AP**

15. (a) (i) Find the Z-transform of $\frac{1}{n(n+1)}$. **(8)** **5** **AP**

(ii) Using the inversion integral method (Residue method), find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$. **(8)** **5** **AP**

(OR)

(b) (i) Use convolution theorem to evaluate $Z^{-1}\left\{\frac{z^2}{(z-a)(z-b)}\right\}$. **(8)** **5** **AP**

(ii) Solve $y(n+2)+6y(n+1)+9y(n)=2^n$ given that $y(0)=0\, and\ y(1)=0$, using Z-transforms. **(8)** **5** **AP**