Q. Code:825724

Reg. No. | | | | | | | | | | | | | |

B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2024

Fourth Semester

MA22454 – PROBABILITY AND QUEUEING THEORY

(Common to CSE and INT)

(Regulation 2022)

TIME: 3 HOURS **MAX. MARKS: 100**

COURSE OUTCOMES	STATEMENT	RBT LEVEL
CO 1	Extend and formalize the knowledge of probability theory and random variables.	**3**
CO 2	Describe commonly used univariate discrete and continuous probability distributions and apply various distributions to solve real world problems.	**3**
CO 3	Identify various distribution functions and acquire skills in handling situations involving more than one variable.	**3**
CO 4	Analyse various classifications of Random Processes and characterize phenomena which evolve with respect to time in a probabilistic manner.	**3**
CO 5	Understand the basic characteristic features of a queuing system and acquire skills in analyzing queuing models.	**3**

PART- A (20 x 2 = 40 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	Let A and B be two events such that $P(A)=\frac{1}{3}, P(B)=\frac{3}{4}, P(A\cap B)=\frac{1}{4}$. Compute $P\left(\frac{A}{B}\right)$ and $P(A\cap\overline{B})$.	**1**	**2**
2.	If X is a continuous random variable with pdf given by $f(x)=Ce^{-\|x\|}; -\infty<x<\infty$, find the value of 'C'.	**1**	**3**
3.	Let X be a random variable with E(X)=10 and Var(X)=25. Find the positive values of a and b such that Y=aX-b has expectation 0 and variance 1.	**1**	**2**
4.	Let $M_X(t)=\frac{1}{1-t}$ such that $t\neq 1$, be the MGF of R.V X. Find the MGF of Y = 2X +1.	**1**	**2**
5.	The number of monthly break down of a computer is a R.V having a poisson distribution with mean equals to 1.8. Find the probability that this computer will function for a month with only one breakdown.	**2**	**2**
6.	If X is a Geometric variable taking values 1,2,3, …, then find P(X is odd).	**2**	**2**
7.	In a small town, there are 900 right-handed individuals and 100 left-handed individuals. We take a sample of size n=20 individuals in this town (random and without	**2**	**2**

	replacement). What is the probability that 4 are more people in the sample are left-handed?		
8.	If X is a normal variate with $mean = 20$ and S.D = 10, find $P[15 \le X \le 40]$.	**2**	**2**
9.	If $f(x,y)=\begin{cases} 8xy, 0<x<1, 0<y<x \\ 0, elsewhere \end{cases}$ is the joint PDF of X & Y, find f(y/x).	**3**	**2**
10.	Determine the value of the constant c if the joint density function of two discrete random variables X and Y is given by $p(x,y)=cxy, x=1,2,3$ and $y=1,2,3$.	**3**	**2**
11.	The correlation coefficient of two random variables X and Y is $\frac{-1}{4}$ while their variances are 3 and 5. Find the covariance.	**3**	**2**
12.	The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the mean of X and Y.	**3**	**2**
13.	Consider the random process $\{X(t)\}, X(t)=y\sin(\omega t)$, where y is uniform in $(-1,1)$. Check whether the process $X(t)$ is WSS or not.	**4**	**3**
14.	A radioactive source emits particles at a rate of 5 per min in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 min period.	**4**	**3**
15.	If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$,find the steady-state distribution of the chain.	**4**	**2**
16.	Consider the Markov chain consisting of the three states A, B, C and transition probability matrix $P=\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$. Draw the state transition diagram.	**4**	**2**
17.	If the arrival and departure rates in (M|M|1) queue are $\frac{1}{2}$ per minute and $\frac{2}{3}$ per minute respectively. find the average waiting time of a customer in the queue.	**5**	**3**
18.	Customers arrive at a one man barber shop according to a Poisson process with a mean inter- arrival time of 12 minute. Customers spend an average of 10 minutes in the barber's chair, what is the expected number of customers in the barber shop?	**5**	**2**
19.	Consider (M|M|C) queueing system. Find the probability that an arriving customer is	**5**	**2**

forced to join the queue.

20. For an (M|M|2): $(\infty \vee FIFO ¿$ queue with $\lambda = \frac{1}{6} / hr, \mu = \frac{1}{4} / hr$, find the probability of an empty system. **5** **2**

PART- B (5 x 10 = 50 Marks)

		Marks	CO	RBT LEVEL
21. (a)	The probability function of an infinite discrete distribution is given by $P(X=x) = \frac{1}{2^x}, x = 1,2,3,\ldots \infty$. Find the mean and variance of the distribution. Also find P(X is even).	**(10)**	**1**	**3**
	(OR)			
(b)	If X is a continuous random variable with pdf given by $f(x) = C e^{-\lvert x \rvert}; -\infty < x < \infty$ Find i) the value of 'C' ii) CDF $F(x)$. iii) MGF and hence find the mean and variance of X.	**(10)**	**1**	**3**
22. (a)	i) 6 dice are thrown 729 times. How many times do you expect atleast three dice to show a five or a six? ii) Boxes contain 200 items of which 10% are defective. Find the probability that no more than 2 defectives will be obtained in a sample of size 10.	**(10)**	**2**	**3**
	(OR)			
(b)	i) State and prove memoryless property of exponential distribution. ii) The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. a) What is the probability that the repair time exceeds 2hrs? b) What is the conditional probability that a repair takes atleast 11 hrs given that its duration exceeds 8 hrs.	**(10)**	**2**	**3**
23. (a)	The joint density function of two random variables X and Y is given by $f(x,y) = \begin{cases} ¿\frac{6}{7}\left(x^2 + \frac{xy}{2}\right), 0 \le x \le 1, 0 \le y \le 2 \\ ¿0, elsewhere \end{cases}$. (a) Compute the marginal density function of X and Y? (b) Find E(X) and E(Y) (c) Find $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$.	**(10)**	**3**	**3**
	(OR)			
(b)	Calculate the correlation coefficient for the following heights in inches of	**(10)**	**3**	**3**

fathers (x) and their sons (y):

x:	65	66	67	67	68	69	70	72
y:	67	68	65	68	72	72	69	71

24. (a) Assume that the number of messages input to a communication channel in an interval of duration t seconds is a Poisson process with mean rate $\lambda = 0.3$. Compute **(10) 4 3**

i) The probability that exactly 3 messages will arrive during a 10 second interval.

ii) The probability that the number of messages arriving in an interval of duration 5 second is between three and seven.

iii) The probability that fewer than 3 messages will arrive during a 12 second interval.

(OR)

(b) A salesman territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either city B or city C, the next day he is twice as likely to sell in city A as in the other city. In the long run how often does he sell in each of the cities? **(10) 4 3**

25. (a) The telephone exchange has two long distance operators. The telephone company finds that during the peak period, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The service time is exponential with a mean of 5 minutes per call. **(10) 5 3**

i) What is the probability that a subscriber will have to wait for his long distance call during peak period?

ii) What is the expected waiting time including service.

(OR)

(b) Consider a single server, Poisson input queue with a mean arrival rate of 10 per hour. Currently, the server works according to an exponential distribution with a mean service time of 5 minutes. Management has a training course after which service time will follow non exponential distribution and the mean service time will increase to 5.5 mins; but the standard deviation will decrease from 5 mins. (exponential case) to 4 mins. Should the server undergo training?. **(10) 5 3**

PART- C (1 x 10 = 10 Marks)

(Q.No.26 is compulsory)

Marks CO RBT LEVEL

26. If X is normally distributed with $\mu = 12$ and $\sigma = 4$. **(10) 2 3**

Find

i) $P(X \geq 20)$

ii) $P(X \leq 20)$

iii) Find a when $P(X > a) = 0.24$.

iv) Find b and c when $P(b<X<c)=0.5 \wedge P(X>c)=0.25$.

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