Q. Code:233060

Reg. No. | | | | | | | | | | | | | |

B.E. / B.TECH. DEGREE

EXAMINATIONS, MAY 2024

Third Semester

MA22355 – PARTIAL DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

(Common to AE, BT and MN)

(Regulation 2022)

TIME: 3 HOURS **MAX. MARKS: 100**

COURSE OUTCOMES	STATEMENT	RBT LEVEL
CO 1	Express proficiency in handling higher order partial differential equations	3
CO 2	Develop skills in classification, formulation, solution, and interpretation of partial differential equations model	3
CO 3	Have the fundamental knowledge of solving an algebraic or transcendental equation, linear system of equations.	3
CO 4	Appreciate the numerical techniques of interpolation in various intervals.	3
CO 5	Solve boundary value problems using finite difference method.	3

PART- A (20 x 2 = 40 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	Form the partial differential equation by eliminating the arbitrary functions from $z=f_1(x)\,f_2(y)$	**1**	**2**
2.	Find the complete integral of $z=px+qy+p^2q^2$	**1**	**2**
3.	Solve $p+q=pq$	**1**	**3**
4.	Solve $(D^3-2D^2D')z=0$	**1**	**3**
5.	If f(x) = 2x in the interval (-4, 4), then find the value of a_n in the Fourier series expansion.	**2**	**2**
6.	Find Fourier sine series expansion of $f(x)=1$ in $(0,\pi)$.	**2**	**2**
7.	The bar of length 10cm has its ends kept at $20^\circ C$ and $60^\circ C$ until steady state condition prevails. Find the steady state temperature of the rod.	**2**	**3**
8.	A rectangular plate is bounded by the line x=0, y=0,x=a, and y=b. Its surfaces are insulated. The temperature along x=0 and y=0 are kept at 0°c and others at $100^\circ c$. Formulate the boundary value problem.	**2**	**3**

9.	Show the Newton Raphson formula for $\sqrt{N}$ is $x_{n+1}=\frac{1}{2}\left(x_n+\frac{N}{x_n}\right), n=0,1,2,3\ldots$	**3**		**2**
10.	Distinguish between direct and iterative methods for solving a system of linear algebraic equations.	**3**		**2**
11.	Solve 3x+y=2, x+3y=-2 using Gauss Elimination method	**3**		**3**
12.	Find the largest eigenvalue of $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ using power method	**3**		**2**
13.	Find the third order divided differences with arguments 2,4,9,10 of the function $f(x)=x^3-2x$	**4**		**2**
14.		4		3

x	0	1	2
y	0	1	20

Using Lagrange's formula, fit a polynomial to the data.

15.	Compare Newton's forward interpolation and Newton's Divided difference interpolation	4		2
16.	Obtain the relation between forward difference operator and shift operator	4		2
17.	Compare Implicit and Explicit method for solving one dimensional heat equation.	5		2
18.	Derive the finite difference form of Poisson equation $\nabla^2 u=y x^2$	5		2
19.	Derive the finite difference form of Laplace equation $\nabla^2 u=0$	5		2
20.	Write down the Crank Nicholson formula to solve $u_{xx}=u_t$ in the general form. When does it assume its simplest form?	5		2

PART- B (5 x 10 = 50 Marks)

		Marks	CO	RBT LEVEL
21. (a)	Solve: $(mz-ny)p+(nx-lz)q=ly-mx$	**(10)**	**1**	**3**
	(OR)			
(b)	Solve: $(D^2-DD'-20\{D')z=e^{5x+y}+\sin(4x-y).¿$	**(10)**	**1**	**3**

22. (a) A tightly stretched string has its ends fixed at $x=0$ and $x=l$ is initially in the position given by $f(x)=k(lx-x^2)$, where k is a constant, and then released from rest. Find the displacement of any point x of the string at anytime $t>0$. **(10) 2 3**

(OR)

(b) Expand $f(x) = x - x^2$ as a Fourier series in $-\pi<x<\pi$. Hence deduce that **(10) 2 3**

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\ldots\ldots\ldots\ldots=\frac{\pi^2}{12}$$

23. (a) Solve using Gauss- Seidel method

$$\begin{aligned} 27x+6y-z&=85\\ x+y+54z&=110\\ 6x+15y+2z&=72 \end{aligned}$$

(10) 3 3

(OR)

(b) Find the numerically largest eigenvalue and the corresponding eigenvector of $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ using the power method. **(10) 3 3**

24. (a) Find the value of y at x=21 and x=28 from the following data using Newton's forward and backward interpolation formula. **(10) 4 3**

x	20	23	26	29
y	0.342	0.3907	0.4384	0.4848

(OR)

(b) Find f(8) by using Newton's divided difference method from the data. **(10) 4 3**

x	3	7	9	10
y=f(x)	168	120	72	63

25. (a) Solve $\nabla^2 u=-10(x^2+y^2+10)$ over the square mesh with sides $x=0, x=3, y=0, y=3$ given $u=0$ on the boundary and mesh length=1 unit. **(10) 5 3**

(OR)

(b)	Solve using Bender-Schmidt method $u_{xx} = u_t$ subject to u (0, t) =0, u (5, t) =0 and $u(x,0) = x^2(25 - x^2) 0 \le x \le 5$, find the value of u up to t=6 seconds by taking h=1.	**(10)**	**5**	**3**

PART- C (1 x 10 = 10 Marks)

(Q.No.26 is compulsory)

		Marks	CO	RBT LEVEL
26.	Using Newton Raphson method find an approximate root of the equation $3x = \cos x + 1$ correct to three decimal places.	**(10)**	**3**	**3**

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