

Reg. No.

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B. E/ B. TECH.DEGREE EXAMINATIONS, MAY 2024

Third Semester

MA22353 – DISCRETE MATHEMATICS*(Computer Science and Engineering and Information Technology)***(Regulation 2022)****TIME: 3 HOURS****MAX. MARKS: 100**

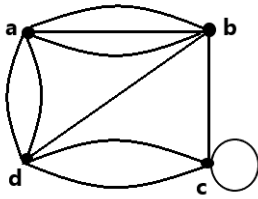
| COURSE OUTCOMES | STATEMENT | RBT LEVEL |
|-----------------|---|-----------|
| CO 1 | Acquire logic to convert from informal language to logic expressions and test the validity of a program.. | 3 |
| CO 2 | Apply the concepts of set theory and counting techniques to comprehend computer simulations. | 3 |
| CO 3 | Develop graph theory tools to map day-to-day applications. | 3 |
| CO 4 | Expose to the concepts and properties of algebraic structures which provide solutions in design and analysis of algorithms. | 3 |
| CO 5 | Explore Boolean algebraic structures on numerous levels, the concepts needed to test the logic of a program. | 3 |

PART- A(20x2=40Marks)

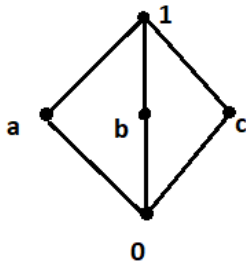
(Answer all Questions)

| | CO | RBT LEVEL |
|--|----|-----------|
| 1. Construct a truth table for the compound proposition $p \oplus (p \wedge q)$. | 1 | 3 |
| 2. Find the bitwise OR and bitwise AND of the pair of bit strings: 1111 1000, 1011 1010. | 1 | 3 |
| 3. What rule of inference is used in the following argument? Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard. | 1 | 2 |
| 4. Let $R(x,y,z)$ denote the statement “ $x + y = z$ ”. What are the truth values of the propositions $R(2,4,6)$ and $R(1,0,0)$? | 1 | 2 |
| 5. Let $f: \{a,b,c,d\} \rightarrow \{1,2,3,4,5\}$ be a function defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$ and $f(d) = 5$. Is f an onto function? Is f one-to-one? | 2 | 3 |
| 6. How many bit strings of length 8 are there? | 2 | 2 |
| 7. A group contains 6 men and 6 women. How many ways are there to arrange these people in a row if the men and women alternate? | 2 | 3 |
| 8. Among any group of 367 people, how many persons will have the same birthday at the minimum? | 2 | 3 |
| 9. How many edges are there in a graph with 5 vertices each of degree 4? | 3 | 2 |
| 10. If G is a simple graph with 15 edges and \bar{G} has 6 edges, then how many vertices does G have? | 3 | 3 |

11. Can a simple graph exist with 5 vertices each of degree 3? 3 2
12. Find the adjacency matrix of the following graph: 3 3



13. Consider the set of natural numbers N under the binary operation $*$ defined by $x * y = \max(x, y), \forall x, y \in N$. Show that $(N, *)$ is a semigroup. 4 2
14. In a group $(G, *)$, prove that $(a^{-1})^{-1} = a, \forall a \in G$. 4 2
15. Find any two cosets of the subgroup $H = \{1, -1\}$ in $G = \{1, -1, i, -i\}$ under the binary operation multiplication. 4 3
16. Find the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication. 4 3
17. Is there a Boolean Algebra with five elements? Justify your answer. 5 2
18. Let (P, \leq) be a poset where $P = \{2, 3, 6, 12, 24, 36\}$ under the relation “division”. Find the upper bounds of $A = \{2, 3, 6\}$ in P . 5 3
19. Consider a relation $R = \{(1, 1), (2, 2)\}$ defined on a set $X = \{1, 2, 3\}$. Is it symmetric? 5 2
20. Find the complements of the elements a, b, c in the following lattice: 5 3



PART- B (5x 10=50Marks)

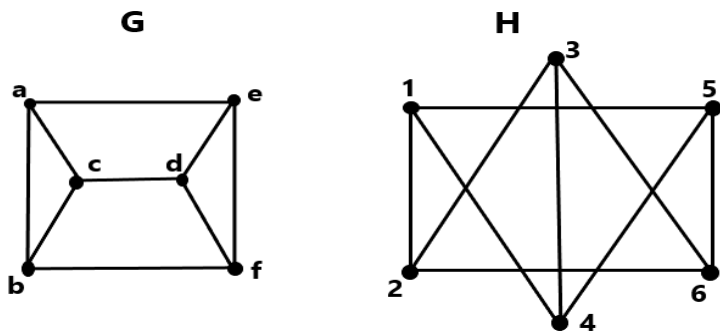
- | | Marks | CO | RBT
LEVEL |
|---|-------|----|--------------|
| 21. (a) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q . | (10) | 1 | 3 |
| (OR) | | | |
| (b) Verify the validity of the following arguments: “All humming birds are richly colored”, “No large birds live on honey”, “Birds that do not live on honey are dull in color”, “Humming birds are small”. | (10) | 1 | 3 |

22. (a) How many positive integers not exceeding 1000 are divisible by any of the integers 3,5,7,11? (10) 2 3

(OR)

- (b) (i) Using mathematical induction, prove that $1^2 - 2^2 + 3^2 - \dots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$ where 'n' is a positive integer. (6) 2 3
- (ii) How many bit strings of length 8 contain (i) exactly four 1s? (ii) utmost four 1s? (iii) at least four 1s? (iv) an equal number of zeros and ones. (4) 2 3

23. (a) Examine whether the following graphs are isomorphic? Justify your answer (10) 3 3



(OR)

(b) Suppose that a company has 4 employees.: Alwin, David, Suman and Thejesh. Suppose that 4 jobs need to be done to complete a project: requirements, architecture, implementation and testing. Suppose that Alwin has been trained to do requirements and testing; David has been trained to do architecture, implementation and testing; Suman has been trained to do requirements, architecture and implementation; Thejesh has only been trained to do requirements

- (a) Model the capabilities of these employees using a bipartite graph.
- (b) Find an assignment of responsibilities such that each employee is assigned a responsibility and no employee is assigned more than one job

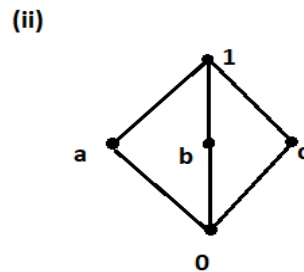
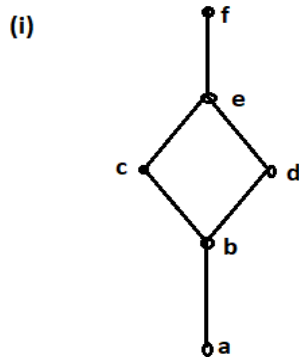
24. (a) Show that \mathbb{Q}^+ is an abelian group where $*$ is defined by $a*b = \frac{ab}{2}, \forall a, b \in \mathbb{Q}^+$. (10) 4 3

(OR)

(b) (i) Find all the subgroups of \mathbb{Q}^+ (6) 4 3

(ii) Let $G = \{1, -1, i, -i\}$ be a group under multiplication. Find the order of its elements. (4)

25. (a) Verify whether the following Hasse Diagrams represent lattices: (10) 5 3



(OR)

(b) Let $D_{42} = \{1, 2, 3, 6, 7, 14, 21, 42\}$ and the relation R be divisor on D_{42} . Find (10) 5 3

- (i) All the lower bounds of 14 and 21.
- (ii) The glb of 14 and 21.
- (iii) All the upper bounds of 14 and 21.
- (iv) The lub of 14 and 21.
- (v) Draw the Hasse diagram

PART- C (1x 10=10Marks)

(Q.No.26 is compulsory)

| Marks | CO | RBT LEVEL |
|-------|----|-----------|
|-------|----|-----------|

26. Find the PDNF and PCNF of the following compound proposition:
 $[P \rightarrow (Q \wedge R)] \wedge [\neg P \rightarrow (\neg Q \wedge \neg R)]$.

| | | |
|------|---|---|
| (10) | 1 | 3 |
|------|---|---|
