**Q. Code:883629**

**Reg. No.** | | | | | | | | | | | | | |

# B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2024

Second Semester

## MA22252 – APPLIED MATHEMATICS II FOR MARINE ENGINEERS

*(Marine Engineering)*

**(Regulation 2022)**

**TIME: 3 HOURS** **MAX. MARKS: 100**

| COURSE OUTCOMES | STATEMENT | RBT LEVEL |
|---|---|---|
| **CO 1** | Apply the basic concepts of ordinary differential equations and its applications in marine engineering problems | **3** |
| **CO 2** | Apply various techniques in solving differential equations. | **3** |
| **CO 3** | Solve gradient, divergence and curl of a vector point function and related identities, evaluation of line, surface and volume integrals using Gauss, Stokes and Green's theorems. | **3** |
| **CO 4** | Recognize fundamental properties of analytic functions and construct simple conformal maps. | **3** |
| **CO 5** | Apply Laplace transforms to solve differential equations. | **3** |

**PART- A (20 x 2 = 40 Marks)**

(Answer all Questions)

| | | CO | RBT LEVEL |
|---|---|---|---|
| **1.** | Determine the order and degree of the differential equation $2023\frac{d^4y}{dx^4}-2024x^{72}\left(\frac{dy}{dx}\right)^8-xy=0$ | **1** | **2** |
| **2.** | Form the differential equation of $y=A\cos 2x+B\sin 2x$ | **1** | **2** |
| **3.** | Solve $(9+x^2)\frac{dy}{dx}=16+y^2$ | **1** | **2** |
| **4.** | Find the integrating factor of $\frac{dy}{dx}+xy=x^2$ | **1** | **2** |
| **5.** | Solve $(D^2+4D+4)\,y=0$ | **2** | **2** |
| **6.** | Find the Particular Integral of $(D^2+1)\,y=e^{2x}$. | **2** | **2** |
| **7.** | Solve $(x^2D^2+xD+1)\,y=0$. | **2** | **2** |
| **8.** | Transform the following equation into a differential equation with constant coefficients $\left[(x+1)^2D^2+(x+1)D+1\right]y=2\sin\left[\log(x+1)\right]$ | **2** | **2** |
| **9.** | Find the unit normal to the surface $x^2+y^2-z=10$ at $(1,2,1)$. | **3** | **2** |
| **10.** | Find $\lambda$ such that $\vec{F}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+\lambda z)\vec{k}$ is solenoidal. | **3** | **2** |
| **11.** | Prove that $\vec{F}=(yz)\vec{i}+(zx)\vec{j}+(xy)\vec{k}$ is irrotational. | **3** | **2** |
| **12.** | Find the normal derivative of $\varphi=xyz^2$ at $(1,0,3)$. | **3** | **2** |

| | | | |
|---|---|---|---|
| 13. | Show that an analytic function with constant real part is constant. | 4 | 2 |
| 14. | Find the invariant points of $f(z)=z^2$ | 4 | 2 |
| 15. | Show that the function $f(z)=\bar{z}$ is nowhere differentiable. | 4 | 2 |
| 16. | Prove that $u=2x-x^3+3xy^2$ is harmonic. | 4 | 2 |
| 17. | Find L $[e^{-2t}\sin 3t]$ | 5 | 2 |
| 18. | Find $L^{-1}\left(\frac{1}{(s+1)^4}\right)$ | 5 | 2 |
| 19. | Verify the initial value theorems for $f(t)=3e^{-2t}$ | 5 | 2 |
| 20. | Find $L^{-1}\left(\frac{4}{(s+2)^2+16}\right)$ | 5 | 2 |

**PART- B (5 x 10 = 50 Marks)**

| | | Marks | CO | RBT LEVEL |
|---|---|---|---|---|
| **21. (a)** | Solve $(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$ | (10) | 1 | 3 |
| | **(OR)** | | | |
| **(b)** | Find the orthogonal trajectory of the cardioids $r=a(1-cos\theta)$ | (10) | 1 | 3 |
| **22. (a)** | Solve $Dx+y=\sin 2t;-x+Dy=\cos 2t$. | (10) | 2 | 3 |
| | **(OR)** | | | |
| **(b)** | Solve $(D^2+a^2)y=\tan ax$ by method of variation of parameter. | (10) | 2 | 3 |
| **23. (a)** | Show that $\vec{F}=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$ is irrotational vector and find its scalar potential. | (10) | 3 | 3 |
| | **(OR)** | | | |
| **(b)** | Verify Green's theorem in the $XY$ plane for $\int_c [(xy+y^2)dx+x^2dy]$ where $C$ is the boundary of the common area between $y=x^2$ and $y=x$. | (10) | 3 | 3 |
| **24. (a)** | Find the image if $\|z-2i\|=2$ under the transformation $w=\frac{1}{z}$ | (10) | 4 | 3 |
| | **(OR)** | | | |
| **(b)** | If $f(z)=u(x,y)+iv(x,y)$ is an analytic function, the curves of the family $u(x,y)=c_1$ and $v(x,y)=c_2$ cut orthogonally, where $c_1$ and $c_2$ are varying constants. | (10) | 4 | 3 |
| **25. (a)** | Find the Laplace transform of the Half-sine wave rectifier function | (10) | 5 | 3 |

$$f(t)=\begin{cases} \sin\omega t \; for \; 0<t<\frac{\pi}{\omega} \\ 0 \; for \; \frac{\pi}{\omega}<t<\frac{2\pi}{\omega} \end{cases}$$

**(OR)**

**(b)** Apply convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. **(10)** **5** **3**

**PART- C (1 x 10 = 10 Marks)**

(Q.No.26 is compulsory)

| | | Marks | CO | RBT LEVEL |
|---|---|---|---|---|
| **26.** | Find the bilinear transformation that maps the points $-2,0,2$ onto $0,i,-i$ respectively. | **(10)** | **4** | **3** |