**Reg. No.**

B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2024
Second Semester
**MA22251 – APPLIED MATHEMATICS II**
(Common to all Branches Except Marine)
**(Regulation 2022)**

TIME: 3 HOURS                                                                                          MAX. MARKS: 100

| CO | STATEMENT | RBT LEVEL |
|---|---|---|
| CO 1 | Interpret the fundamentals of vector calculus and execute evaluation of line, surface and volume integrals using Gauss, Stokes, and Green's theorems. | 3 |
| CO 2 | Solve first order linear, homogeneous differential equations and use series solution method to solve second order differential equations. | 3 |
| CO 3 | Determine the methods to solve differential equations using Laplace transforms and Inverse Laplace transforms. | 3 |
| CO 4 | Explain Analytic functions and Categorize transformations. | 3 |
| CO 5 | Perform Complex integration to evaluate real definite integrals using Cauchy integral theorem and Cauchy's residue theorem. | 3 |

**PART- A (20 x 2 = 40 Marks)**
(Answer all Questions)

|  |  | CO | RBT LEVEL |
|---|---|---|---|
| 1. | Find the values of $a,b,c$ so that $A=(x+y+az)\vec{i}+(bx+2y-z)\vec{j}+(-x+cy+2z)\vec{k}$ is irrotational. | 1 | 2 |
| 2. | Find a unit normal to the surface $x^2 y+2xz^2=8$ at the point $(1,0,2)$. | 1 | 3 |
| 3. | Prove that $\vec{F}=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ is a conservative force field. | 1 | 3 |
| 4. | Evaluate $\iint\limits_{S} \vec{r}.\hat{n}\,ds$, where S is a closed surface. | 1 | 2 |
| 5. | Find the particular integral of $(D^2+1)\,y = \cos(2x-1)$. | 2 | 2 |
| 6. | Transform the equation $(x^2 D^2 + x D)\,y = x$ into a differential equation with constant coefficients. | 2 | 2 |
| 7. | Solve $\dfrac{dy}{dx}=\dfrac{1+y^2}{1+x^2}$. | 2 | 3 |
| 8. | Check whether the differential equation is exact or not $(5x^4+3x^2 y^2-2xy^3)dx+(2x^3 y-3x^2 y^2-5y^4)dy=0$. | 2 | 2 |
| 9. | Give two examples for which the Laplace transform does not exist. | 3 | 2 |
| 10. | Find Laplace Transform of $e^{-5t}t^7$ | 3 | 2 |
| 11. | If $L(f(t))=\dfrac{s+2}{s^2+4}$, find $\int\limits_{0}^{\infty} f(t)dt$. | 3 | 3 |

| | | | |
|---|---|---|---|
| 12. | If $L(f(t)) = \dfrac{1}{s(s+a)}$, find $f(0)$ & $f(\infty)$ | 3 | 3 |
| 13. | Check whether $f(z)$ ¿$e^x$ (cos $y$ $-$ ¿ $i$ sin $y$) is analytic or not. | 4 | 2 |
| 14. | Check whether the function $u = 2x - x^3 + 3xy^2$ is harmonic or not. | 4 | 2 |
| 15. | Find the image of $|z - 2i| = 2$ under the mapping $w = \dfrac{1}{z}$ . | 4 | 2 |
| 16. | Find the fixed points of the transformation $w = \dfrac{z-1}{z+1}$ . | 4 | 2 |
| 17. | Evaluate $\displaystyle\oint_c \dfrac{dz}{z+4}$, where $C$ is the circle $|z| = 2.$ | 5 | 2 |
| 18. | Expand $\dfrac{1}{z-2}$ at $z$ ¿ $1$ in Taylor's series. | 5 | 2 |
| 19. | Find the nature of the singular points of $\dfrac{e^z}{z^2+4}.$ | 5 | 2 |
| 20. | Find the residue of $\dfrac{1-e^{2z}}{z^4}$ at $z = 0.$ | 5 | 2 |

## PART- B (5 x 10 = 50 Marks)

| | | Marks | CO | RBT LEVEL |
|---|---|---|---|---|
| 21. (a) | Find the directional derivative of $xy^2 + yz^3$ at $(2,-1,1)$ in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at $(-1,2,1)$ . | (10) | 1 | 3 |
| | **(OR)** | | | |
| (b) | Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\,\vec{j}$ in the rectangular region in the xy plane bounded by the lines $x = 0, x = a, y = 0, y = b.$ | (10) | 1 | 3 |
| 22. (a) | Solve $(D^2 - 3D + 2)\, y = 2\cos(2x+3) + 2e^x$ . | (10) | 2 | 3 |
| | **(OR)** | | | |
| (b) | Solve $[(1+x)^2 D^2 + (1+x)D + 1]\, y = 2 \sin[\log(1+x)].$ | (10) | 2 | 3 |
| 23. (a) | Find the Laplace transform of $f(t) = \begin{cases} a \sin \omega t, 0 \le t \le \pi/\omega \\ 0, \pi/\omega \le t \le 2\pi/\omega \end{cases}$ and $f(t + 2\pi/\omega) = f(t)$ . | (10) | 3 | 3 |
| | **(OR)** | | | |
| (b) | Find the inverse Laplace transform of $\dfrac{s}{(s^2 + a^2)^2}$ using convolution theorem. | (10) | 3 | 3 |
| 24. (a) | If $u = \log(x^2 + y^2)$, then find $v$, and $f(z)$ such that $f(z) = u + iv$ is analytic. | (10) | 4 | 3 |
| | **(OR)** | | | |
| (b) | Find the image of the infinite strip $\dfrac{1}{4} \le y \le \dfrac{1}{2}$ under the transform w = 1/z. | (10) | 4 | 3 |

25. (a) Using Cauchy's integral formula, evaluate $\displaystyle\int_C \frac{dz}{(z+1)^2(z-2)}$ , **(10)** **5** **3**

where C is $|z|=\dfrac{3}{2}$.

(OR)

(b) Using Cauchy's residue theorem, evaluate $\displaystyle\int_c \frac{z-1}{(z-2)(z+1)^2}\,dz$ , **(10)** **5** **3**

where C is $|z-i|=2$.

**PART- C (1 x 10 = 10 Marks) (Q.No.26 is compulsory)**

| | Marks | CO | RBT |
|---|---|---|---|

26. Prove that $(y^2\cos x+z^3)\vec{i}+(2\,ysinx-4)\vec{j}+3\,x\,z^2\vec{k}$ is irrotational and find its scalar **(10)** **1** **3**
potential.