Q. Code:789333

Reg. No. | | | | | | | | | | | | | |

B. E / B. TECH.DEGREE EXAMINATIONS, MAY 2024

Fourth Semester

MA18454 – PROBABILITY AND RANDOM PROCESSES

(Electronics and Communication Engineering)

(Regulation 2018 /2018A)

TIME:3 HOURS **MAX. MARKS: 100**

CO1 Reproduce and explain the basic concepts such as probability and random variable and identify the distribution.

CO2 Acquire skills in handling situations involving more than one random variable.

CO3 Study the characterize phenomena with respect to time in probabilistic manner.

CO4 Apply the relationship within and between random processes.

CO 5 Apply the response of random inputs to linear time invariant systems.

PART- A(10x2=20Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	A continuous random variable X has probability density function f(x) = k (1+x), in 2 < x <5. Find P(3 <X <4).	1	2
2.	The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.	1	2
3.	The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = k(x^3y + xy^3)$, $0 \le x \le 2 ; 0 \le y \le 2$. Find the value of k .	2	2
4.	If $F(x,y)=$¿$(1-x^2)(1-y^2)$ if $x,y\geq 0$¿¿¿¿, then find the joint probability density function.	2	2
5.	If the random process $X(t)=\cos(\omega t+\theta)$ where θ *is*uniformly distributed in the interval $(-\pi,\pi)$.Find E[X (t)].	3	2
6.	State the four types of Stochastic Processes.	3	1
7.	Given the power spectral density $S_{XX}(\omega)=\frac{1}{4+\omega^2}$, find the average power of the process.	4	2

			Marks	CO
8.	The power spectral density of a random process X(t) is given by $S_{XX}(\omega)=\{\pi, \lvert\omega\rvert<1 \wedge ¿ 0, otherwise$ Find its autocorrelation.		**4**	**2**
9.	Prove that system y(t) = t x(t) – x (t – 1) is time invariant.		**5**	**2**
10.	Define a system. When it is called a linear system?		**5**	**2**

PART- B (5x 14=70Marks)

			Marks	CO	RBT LEVEL
11. (a)	**(i)**	For the triangular distribution $f(x)=\begin{cases} x, 0<x\le 1 \\ 2-x, 1\le x<2 \\ 0, otherwise \end{cases}$, find the mean, variance and the moment generating function.	**(7)**	**1**	**3**
	(ii)	The time (in hours) required to repair a machine is exponentially distributed with parameter λ = 1/2. What is the probability that the repair time exceeds 2hours? What is the conditional probability that a repair takes at least 10hours given that its duration exceeds 9hours?	**(7)**	**1**	**3**

(OR)

(b) **(i)** A discrete random variable X has the following probability distribution. **(7)** **1** **3**

X	0	1	2	3	4	5	6	7	8
P(X = x)	a	3a	5a	7a	9 a	11a	13 a	15a	17a

Find (i) the value of ‘a’ (ii) P(X<3) (iii) P(0<X<3) and (iv) P(X≥3).

(ii) Trains arrive at a station at 15-minute intervals starting at 4 am. If a passenger arrives at the station at a time that is uniformly distributed between 9.00 and 9.30 am, find the probability that he has to wait for the train for (i) less than 6 minutes and (ii) more than 10 minutes. **(7)** **1** **3**

12. (a) Find the correlation coefficient ρ_{xy} for the following joint density function **(14)** **2** **3**

$f(x,y)=¿(x+y, 0<x<1, 0<y<1¿¿¿¿$

(OR)

(b)	The joint probability mass function of (X,Y) is given by P(x, y) = k(2x+3y), x = 0,1,2; y = 1,2,3. Find all marginal probability distributions. Also find the probability distribution of (X+Y).	**(14)**	**2**	**3**
13. (a)	Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes (i) exactly 4 customers arrive (ii) greater than 4 customers arrive (iii) fewer than 4 customers arrive and (iv) no customers arrive.	**(14)**	**3**	**3**

(OR)

(b)	Suppose X(t) is a normal process with mean $\mu(t)=3$ and $C(t_1,t_1)=4e^{-0.2\|t_1-t_2\|}$. Find (i) $P[X(5)\le 2]$ and (ii) $P[\|X(8)-X(5)\|\le 1]$.	**(14)**	**3**	**3**
14. (a) (i)	If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases} ¿\frac{b}{a}(a-\|\omega\|);\|\omega\|\le a \\ ¿0;\|\omega\|>a\end{cases}$, find its autocorrelation function of the process.	**(7)**	**4**	**3**
(ii)	If {X(t)} is a WSS process with autocorrelation $R_{XX}(\tau)$ and if $Y(t)=X(t+a)-X(t-a)$. Show that $R_{YY}(\tau)=2R_{XX}(\tau)-R_{XX}(\tau+2a)-R_{XX}(\tau-2a)$.	**(7)**	**4**	**3**

(OR)

(b)	Consider two random processes $X(t)=3\cos(\omega t+\theta)$and $Y(t)=2\cos\left(\omega t+\theta-\frac{\pi}{2}\right)$, where θ is a random variable uniformly distributed in$(\theta,2\pi)$. Prove that$\|R_{XY}(\tau)\|\le\sqrt{R_{XX}(0)R_{YY}(0)}$.	**(14)**	**4**	
15. (a)	If$\{N(t)\}$ is a band limited White noise with a power spectral density $S_{NN}(\omega) = ¿\begin{cases}\frac{N_0}{2}, for\ \|\omega-\omega_0\|<\omega_B ¿¿¿\end{cases}$. Find the autocorrelation of $\{N(t)\}$.	**(14)**	**5**	**3**

(OR)

(b)	A circuit has unit impulse response given by	**(14)**	**5**	**3**

$h(t)=\begin{cases} \frac{1}{T}; & 0 \le t \le T \\ 0; & elsewhere \end{cases}$. Evaluate $S_{YY}(\omega)$ interms of $S_{XX}(\omega)$

PART- C (1x 10=10Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16.	Two regression lines are 4x-5y + 33 = 0 and 20x - 9y = 107. Find the means of x and y. Also find the value of the correlation coefficient.	**(10)**	**2**	**3**

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