Q. Code:580756

Reg. No. | | | | | | | | | | | | | |

B.E. / B.TECH. DEGREE EXAMINATIONS, MAY 2024

Fourth-Semester

MA18453 – PROBABILITY AND QUEUING THEORY

(Common to CSE and INT)

(Regulation 2018/2018A)

TIME: 3 HOURS **MAX. MARKS: 100**

COURSE OUTCOMES	STATEMENT	RBT LEVEL
CO 1	Describe commonly used univariate discrete and continuous probability distributions by formulating fundamental probability distributions and density functions, as well as functions of random variables.	**3**
CO 2	Develop skills in dealing with scenarios involving multiple random variables.	**3**
CO 3	Express and characterize phenomenon which evolve with respect to time in a probabilistic manner.	**3**
CO 4	Acquire skills in analyzing queueing models.	**3**
CO 5	Develop skills in identifying best techniques to solve a specific problem.	**3**

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	If $f(x)=kx^2,\ 0<x<3$ is to be a density function, find the value of k.	**1**	**2**
2.	A discrete random variable X has the moment generating function $M_X(t)=\left(\frac{1}{4}+\frac{3}{4}e^t\right)^5$. Find E(X) and Var(X).	**1**	**2**
3.	The correlation coefficient of two random variables X and Y is $\frac{-1}{4}$ while their variances are 3 and 5. Find the covariance.	**2**	**2**
4.	The regression equation of X on Y and Y on X are respectively $5X-Y=22, 64X-45Y=24$. Find the means of X and Y.	**2**	**2**
5.	Consider the random process $\{X(t)\}, X(t)=\cos(t+\phi)$, where ϕ is uniform in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$. Check whether the process is stationary or not.	**3**	**2**
6.	The one step tpm of a Markov chain with states 0 and 1 is given as $P=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. Draw the transition diagram.	**3**	**2**
7.	What is the probability that a customer has to wait more than 15 minutes to get the service completed in (M/M/1): (∞/FIFO) queueing system if λ=6/hr and μ=10/hr?	**4**	**2**

8. Find the values of P_0 and P_n for the queuing model (M/M/1): (K/FIFO) when $\lambda=\mu$. **4** **2**

9. An M/D/M queue has an arrival rate of 10 customers per second and a service rate of 20 customers per second. Compute the mean number of customers in the system. **5** **2**

10. Consider a service facility with two sequential stations with respective service rates of 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system, if the system could be approximated by a two-stage Tandem queue? **5** **2**

PART- B (5 x 14 = 70 Marks)

Marks **CO** **RBT LEVEL**

11. (a) **(i)** A discrete random variable X has the following probability distribution: **(8)** **1** **3**

Find the

x	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

value of 'a', $P(X<3)$, $P(X\geq 3)$ and $P(0<X<3)$.

(ii) The distribution function of a random variable X is given by $F(x)=1-(1+x)e^{-x},\ x\geq 0$. Find the density function, mean and variance of X. **(6)** **1** **3**

(OR)

(b) **(i)** Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7, what is the probability that (a) the target would be hit on the tenth attempt?
(b) it takes him less than 4 shots to hit the target? (c) it takes him an even number of shots to hit the target? **(7)** **1** **3**

(ii) The annual rainfall in inches in a certain region has a normal distribution with mean 40 and variance 60. What is the probability that the rainfall in a given year is between 30 and 48 inches? **(7)** **1** **3**

12. (a) The joint probability distribution of (X, Y) is given by **(14)** **2** **3**

Y / X	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Find (a) the conditional distribution of X given Y=y_j (b) the conditional distribution of Y given X = x_i (c) P(X¿2) (d)$P(Y\leq 3)$
(e) P(X+Y< 4) (f)Verify whether X and Y are independent or not.

(OR)

Q. No.		Question	Marks		
(b)		Two random variables X and Y have the joint pdf $f(x,y)=¿\begin{cases}\frac{x+y}{3}, & 0<x<1,\ 0<y<2¿¿¿\end{cases}$. Find the correlation co-efficient. Also find the equations of the two regression lines.	**(14)**	**2**	**3**
13. (a)	**(i)**	The customers arrive at a bank according to Poisson process with a mean rate of 2 per minute. Find the probability that the interval between 2 consecutive arrivals is (a) more than one minute (b) between 1 and 2 minutes (c) less than 4 minutes**.**	**(7)**	**3**	**3**
	(ii)	Verify whether the process $X(t)=Y\sin(\omega t), Y\sim U(-1,1)$ is WSS or not.	**(7)**	**3**	**3**
		(OR)			
(b)	**(i)**	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared, find (a) the probability that he takes a train on the third day and (b) the probability that he drives to work in the long run.	**(7)**	**3**	**3**
	(ii)	Three girls A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition probability matrix and classify the states.	**(7)**	**3**	**3**
14. (a)		Customers arrive at one – man barber shop according to Poisson process with a mean inter arrival of 12 min. Customers spend an average of 10 min. in the barber's chair. (a) What is the expected number of customers in the barber shop and in the queue? (b) How much time can a customer expect to spend in the barber shop? (c) What is the average time a customer spends in the queue? (d) What is the probability that the waiting time in the system is greater than 30 min. (e) Calculate the percentage of customers who have to wait prior to getting into the barber's chair. (f) What is the probability that more than 3 customers are in the system? (g) Management will provide another chair and hire another barber when a customer's waiting time in the shop exceeds 1.25 (h). How much must the average rate of arrivals increase to warrant a second barber?	**(14)**	**4**	**3**
		(OR)			

(b) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. **(14)** **4** **3**

(a) Find the effective arrival rate at the clinic.

(b) What is the probability that an arriving patient will not wait?

(c) What is the expected waiting time until a patient is discharged from the clinic?

15. (a) A car wash facility operates with only one bay. Cars arrive according to Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car has uniform distribution between 8 and 12 minutes. **(14)** **5** **3**

(OR)

(b) In a book-shop there are 2 sections, one for text books and the other for note books. Customers from outside arrive at the text book section at a Poisson rate of 4/hr and at the note book section at a Poisson rate of 3/hr. The service rates of the T.B. section and N.B. section are respectively 8 and 10/hr. A customer upon completion of service at T.B. section is equally likely to go to the N.B. section or to leave the book shop, whereas a customer upon completion of service at N.B. section will go to the T.B. section with probability 1/3 and will leave the book shop otherwise. Find the joint steady state probability that there are 4 customers in the T.B. section and 2 customers in the N.B. section. Find also the average number of customers in the book shop and the average waiting time of a customer in the shop. (Assume that there is only one salesman in each section) **(14)** **5** **3**

PART- C (1 x 10 = 10 Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16.	The number of personal computer (PC) sold daily at a showroom is uniformly distributed with a minimum of 2000 PC and a maximum of 5000 PC. Find a) The probability that daily sales will fall between 2500 and 3000 PC's. b) What is the probability that the showroom will exactly sell 2500's?	**(10)**	**1**	**3**

c) What is the probability that the showroom will sell at least 4000 PC's?

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