Q. Code:277816

Reg. No. | | | | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2024

Fourth Semester

MA18452 – PARTIAL DIFFERENTIAL EQUATIONS AND COMPUTATIONAL METHODS

(Automobile Engineering)

(Regulation 2018 / 2018A)

TIME: 3 HOURS **MAX. MARKS: 100**

CO 1 Students will be able to identify and solve partial differential equations analytically.
CO 2 Students will be familiar with the application of the Fourier series concept in Boundary value problems.
CO 3 Students will be familiar with the techniques of solving algebraic or transcendental equations and linear system of equations.
CO 4 Students will acquire the knowledge of Interpolation and Approximation and Curve fitting.
CO 5 Students will be aware of solving partial differential equations numerically.

PART- A (10 x 2 = 20 Marks)

(Answer all Questions)

		CO	RBT LEVEL
1.	Form partial differential equation by eliminating arbitrary constants a and b from $z=ax+by+ab$.	**1**	**2**
2.	Solve $p+q=1$.	**1**	**3**
3.	Classify the partial differential equation $u_{xx}+3u_{xy}+4u_{yy}+u_x=0$.	**2**	**2**
4.	The ends A and B of a rod 20 cm long have the temperature at $30^\circ C$ and $80^\circ C$ until steady state conditions prevail. Find the steady state temperature.	**2**	**2**
5.	Find an iterative formula to find $\sqrt{N}$, where N is a positive number by Newton-Raphson method.	**3**	**2**
6.	State the sufficient condition for Gauss-Jacobi method to converge.	**3**	**1**
7.	What is the Lagrange's formula to find y if three sets of values $(x_0, y_0), (x_1, y_1)$, and (x_2, y_2) are given?	**4**	**1**
8.	When Newton's forwarded interpolation formula is used?	**4**	**2**
9.	Write the finite difference form of $u_{xx}+u_{yy}=0$.	**5**	**1**
10.	Write an explicit formula to solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ numerically.	**5**	**1**

PART- B (5 x 14 = 70 Marks)

			Marks	CO	RBT LEVEL
11. (a)	**(i)**	Form partial differential equation by eliminating arbitrary functions f and φ from $z=f(x+ct)+\varphi(x-ct)$.	**(7)**	**1**	**3**
	(ii)	Solve $(y-z)p-(2x+y)q=2x+z$.	**(7)**	**1**	**3**
		(OR)			
(b)	**(i)**	Find the singular solution of $z=px+qy-2\sqrt{pq}$.	**(7)**	**1**	**3**
	(ii)	Solve $(D^3-2D^2D')z=2e^{2x}+3x^2y$.	**(7)**	**1**	**3**
12. (a)		A tightly stretched string with fixed end points $x=0$ and $x=100$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(100-x)$, find the displacement of any point on the string at a distance from one end at any time t.	**(14)**	**2**	**3**
		(OR)			
(b)		An infinitely long rectangular plate is of width 10 cm. The temperature along the short edge y = 0 is given by $u=\begin{cases}20x & \text{for } 0\le x\le 5\\ \end{cases}$. If all the other three edges are kept at zero temperature, find the steady state temperature at point on it.	**(14)**	**2**	**3**
13. (a)	**(i)**	Find by Newton-Raphson method, the root of the equation $e^x=4x$, correct to three places of decimals.	**(7)**	**3**	**3**
	(ii)	Solve the system of equations $x+y+54z=110, 27x+6y-z=85, 6x+15y+2z=72$ using Gauss-Seidel iteration method.	**(7)**	**3**	**3**
		(OR)			
(b)	**(i)**	Solve the system of equations $x+3y+3z=16, x+4y+3z=18, x+3y+4z=19$ using Gauss-Jordan method.	**(7)**	**3**	**3**
	(ii)	Using Power method to find the dominant Eigenvalue and the	**(7)**	**3**	**3**

corresponding Eigenvector of the matrix $A=\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$.

14. (a) **(i)** Using Lagrange's interpolation formula, find the value corresponding to $x=10$ from the following table: **(7)** **4** **3**

x	5	6	9	11
y	12	13	14	16

(ii) A third degree polynomial passes through $(0,1),(1,-1)(2,-1)\wedge(3,2)$. Find its value at $x=4$ using Newton's forward interpolation formula. **(7)** **4** **3**

(OR)

(b) **(i)** Find the cubic function from the following table using Newton's divided difference formula. **(7)** **4** **3**

x	0	1	3	4
$f(x)$	1	4	40	85

(ii) Apply Newton's backward formula to find a polynomial of degree three which includes the following values of x and y. **(7)** **4** **3**

x	3	4	5	6
y	6	24	60	120

15. (a) Solve the Poisson's equation $\nabla^2 u=-10(x^2+y^2+10)$ over the square mesh with sides $x=0,\ y=0,\ x=3,\ y=3$ with $u=0$ on the boundary and mesh length 1 unit. **(14)** **5** **3**

(OR)

(b) **(i)** Solve by Crank-Nicolson's method $\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2}, 0<x<1, t>0,$ **(7)** **5** **3**

$u(x,0)=u(0,t)=0, u(1,t)=100t$. Compute u for one step with $h=\frac{1}{4}$.

(ii) Solve $4\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$ with boundary conditions **(7) 5 3**

$u(0,t)=u(4,t)=0$, $u(x,0)=x(4-x)$ and $u_t(x,0)=0$.

PART- C (1 x 10 = 10 Marks)

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16.	Using Gauss-Jordan method find the inverse of the matrix $A=\begin{pmatrix}2 & 2 & 3\\ 2 & 1 & 1\\ 1 & 3 & 5\end{pmatrix}$.	**(10)**	**3**	**3**

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