

Reg. No.

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**B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2024**

Third Semester

**MA18351 – ENGINEERING MATHEMATICS-III**

(Common to All Branches except MR)

*(Regulation 2018 & 2018A)***TIME: 3 HOURS****MAX. MARKS: 100**

- CO 1** Express proficiency in handling higher order Partial differential equations.
- CO 2** Acquire the skill in examining a signal in another domain rather in the original domain by handling Full and Half Range Fourier Series.
- CO 3** Develop skills in classification, formulation, solution, and interpretation of PDE models.
- CO 4** Develops the skill of conversion between time domain to frequency domain using the concept of Fourier Transforms.
- CO 5** Apply the systematic method for finding the impulse response of LTI systems described by difference equations: partial fraction expansion.

**PART- A (10 x 2 = 20 Marks)**

(Answer all Questions)

		CO	RBT LEVEL
1	Form the PDE of the following by eliminating arbitrary constants from $(x-a)^2+(y-b)^2=z^2 \cot^2 \alpha$	1	2
2	Solve $\sqrt{p}+\sqrt{q}=1$	1	2
3	Determine the value of $a_n$ in the Fourier series expansion of $f(x)=x^3$ in $-\pi < x < \pi$ .	2	2
4	Find the root mean square value of $f(x)=x^2$ in the interval $(0, \pi)$ .	2	2
5	The ends A and B of a rod of length 10cm long have their temperature kept at $20^\circ \text{C}$ and $70^\circ \text{C}$ . Find the steady state temperature distribution on the rod.	3	2
6	How many conditions are required to solve $u_t = \alpha^2 u_{xx}$ .	3	1
7	If $F[f(x)] = F(s)$ , prove that $F[f(ax)] = \frac{1}{ a } F\left(\frac{s}{a}\right)$ .	4	1
8	Find the Fourier sine transform of $3e^{-2x}$ .	4	2
9	Evaluate $Z^{-1} \left[ \frac{z}{z^2+7z+10} \right]$ .	5	2
10	Form difference equation from $U_n = a2^{n+1}$ .	5	2

**PART- B (5 x 14 = 70 Marks)**  
(Restrict to a maximum of TWO subdivisions)

		Marks	CO	RBT LEVEL														
<b>11(a)</b>	<b>(i)</b> Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$	(7)	1	3														
	<b>(ii)</b> Solve $x(y-z)p + y(z-x)q = z(x-y)$	(7)	1	3														
	<b>(OR)</b>																	
<b>11(b)</b>	<b>(i)</b> Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial xy} - 6 \frac{\partial^2 z}{\partial y^2} = \cos(2x+y) + e^{x-y}$	(7)	1	3														
	<b>(ii)</b> Form the PDE of the following by eliminating arbitrary functions from $\varphi\left[z^2 - xy, \frac{x}{z}\right] = 0$	(7)	1	3														
<b>12(a)</b>	<b>(i)</b> Find the Fourier series expansion of $f(x) = \begin{cases} \pi x & \text{in } 0 \leq x \leq 1 \\ 0 & \text{in } 1 < x \leq 2 \end{cases}$ in the interval (0, 1).	(7)	2	3														
	<b>(ii)</b> Find the Fourier series expansion of $f(x) = x^2$ in $-\pi < x < \pi$ . Hence prove $\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots$	(7)	2	3														
	<b>(OR)</b>																	
<b>12(b)</b>	<b>(i)</b> Find the half range cosine series of $f(x) = (x-2)^2$ in $0 < x < 2$ .	(7)	2	3														
	<b>(ii)</b> Find the Fourier series up to second harmonic from the following data.	(7)	2	3														
	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>9</td> <td>18</td> <td>24</td> <td>28</td> <td>26</td> <td>20</td> </tr> </table>	x	0	1	2	3	4	5	y	9	18	24	28	26	20			
x	0	1	2	3	4	5												
y	9	18	24	28	26	20												
<b>13(a)</b>	A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l-x)$ , find the displacement of any point on the string at a distance from one end at any time t.	<b>(14)</b>	3	3														
	<b>(OR)</b>																	
<b>13(b)</b>	An infinitely long rectangular plate with insulated surfaces is 10 cm wide. The two long edges and one short edge are kept at zero temperature while the other short edge $x=0$ is kept at temperature.	<b>(14)</b>	3	3														

$$u(x,t) = 20y \text{ for } 0 \leq y \leq 5$$

Find the steady state temperature distribution in the plate.

14(a) Find the Fourier transform of  $f(x)$  defined as (14) 4 3

$$f(x) = a - |x|, \text{ for } |x| < a$$

Hence show that  $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$  and  $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ .

(OR)

14(b) (i)  $f(x) = x, \text{ for } 0 < x < 1$  and  $f(x) = 2-x, \text{ for } 1 < x < 2$  (7) 4 3

Find the Fourier sine transforms of

(ii)  $\int_0^{\infty} \frac{dx}{(a^2+x^2)(b^2+x^2)}$  (7) 4 3

Using Fourier Transform technique evaluate

15(a) (i)  $\frac{1}{(n+1)(n+2)}$  (7) 5 3

Find the Z-transform of

(ii) Using Convolution theorem find inverse Z-transforms of (7) 5 3

$$\frac{z^2}{(z-a)(z-b)}$$

(OR)

15(b) Solve the following difference equations using z-transforms (14) 5 3

$$u_{n+2} + 4u_{n+1} + 3u_n = 3^n \text{ With } u_0 = 0, u_1 = 1.$$

**PART- C (1 x 10 = 10 Marks)**

(Q.No.16 is compulsory)

16  $\frac{z^3 + 3z}{(z-1)^2(z^2+1)}$  Marks (10) CO 5 RBT LEVEL 3

Using partial fractions find inverse Z-transforms of

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