Q. Code:707335

Reg.No. | | | | | | | | | | |

B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2024

Second Semester

MA18253 - ALGEBRA FOR DATA SCIENCE

(Artificial Intelligence and Data Science)

(Regulation2018 & 2018A)

TIME:3 HOURS **MAX.MARKS: 100**

CO1	Acquire the logical and mathematical maturity to deal with abstraction
CO2	Explore the notion of proof and apply the same in real life problems
CO3	Acquire the knowledge in concepts and properties of algebraic structures and their applications.
CO4	Develops the skill to apply the concepts of Polynomials in Rings and Fields.
CO5	Explore the concepts and significance of lattices and Boolean algebra which are widely used in data science

PART- A(10x2=20Marks)

(Answer all Questions)

		Marks	CO	RBT LEVEL
1	Write the truth table for $\neg p \rightarrow q$	**2**	**1**	**2**
2	Negate the statement "CSK won the trophy in IPL 2023 " in two ways.	**2**	**1**	**2**
3	What is the truth value of P(-2) when P(x): x < 5 , the universe of discourse is the set of positive integers.	**2**	**2**	**2**
4	Express the statement ' For every 'x' there exists a 'y' such that $x^2 + y^2 \geq 100$" in symbolic form.	**2**	**2**	**2**
5	What is the inverse of 3 in the group {1,3,5,7} under multiplication modulo 8?	**2**	**3**	**2**
6	Find the identity element of the set of integers with the binary operation defined by a*b = a+b +3ab, $\forall a, b \in Z$.	**2**	**3**	**2**
7	Find the roots of the polynomial $x^2 + 6$ over the field C.	**2**	**4**	**2**
8	What is the remainder when $f(x) = 2x^3 - 3x^2 + 2x - 9 \in Z_5[x]$ is divided by $x - 2$.	**2**	**4**	**2**
9	Draw the Hasse diagram of D_{24} = {1, 2, 3, 4, 6, 8,12, 24}.	**2**	**5**	**2**
10	Can a poset (D_{30}, |) form a finite Boolean algebra?	**2**	**5**	**2**

PART- B (5x 14=70Marks)

Q.No.		Question	Marks	CO	RBT LEVEL
11(a)	**(i)**	Verify whether $[(q \vee r) \to (p \wedge \neg q)]$ is a tautology.	7	1	3
	(ii)	Obtain the PDNF and PCNF of $(\neg p \wedge q) \wedge (q \to p)$	7	1	3
		(OR)			
11(b)	**(i)**	Show that $(P \vee Q) \to R \Rightarrow (P \to R) \wedge (P \to Q)$	7	1	3
	(ii)	Use the indirect method to show that $p \to q, q \to r, \ p \vee r \Rightarrow r$.	7	1	3
12(a)	**(i)**	Show that $(x)[P(x) \to Q(x)] \wedge (x)[Q(x) \to R(x)] \Rightarrow (x)[P(x) \to R(x)]$	7	2	3
	(ii)	Use rules of inference to prove that the premises "A student in this class has not read the book" and "Everyone in the class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book".	7	2	3
		(OR)			
12(b)	**(i)**	Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ by indirect method.	7	2	3
	(ii)	Prove that if x and y are rational numbers then x+y is a rational number by direct method	7	2	3
13(a)	**(i)**	Show that the set Q^+ of all positive rational numbers forms an abelian group under the operation * defined by; a ,b $a * b = \frac{ab}{2} a, b \in Q^+$	8	3	3
	(ii)	Determine $(i)\ \alpha\beta \quad (ii)\ \alpha^{-1}\ and\ \beta^{-1}\ (iii)\ (\alpha\beta)^{-1}$ In a group S_5={1,2,3,4,5} where $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ & $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 3 & 4 \end{pmatrix}$	6	3	3
		(OR)			
13(b)	**(i)**	State and Prove Lagrange's theorem	8	3	3
	(ii)	Find all the cosets of $\{[0],[3]\}$ in the group of $(Z_9, +_9)$	6	3	3

			Marks	CO	RBT LEVEL
14(a)	**(i)**	Determine whether the polynomial $f(x) = x^2 + 4x \in Z_{12}[x]$ is irreducible over Z_{12}	**7**	**4**	**3**
	(ii)	Find $[17]^{-1}$ in the ring Z_{1009}	**7**	**4**	**3**
		(OR)			
14(b)	**(i)**	Find the g. c. d of $x^4 + x^3 + 1$ and $g(x) = x^2 + x + 1$ over $Z_2[x]$	**7**	**4**	**3**
	(ii)	Determine all the roots of the polynomial $f(x) = x^2 + x + 4 \in Z_{11}[x]$	**7**	**4**	**3**
15(a)	**(i)**	Let S_{30} be the set of positive divisors of 30. If $\leq$ is the relation of divisibility, prove that $(S_{30}, \leq)$ is a Lattice. Draw the Hasse diagram of it.	**7**	**5**	**3**
	(ii)	Prove that every chain is a distributive Lattice.	**7**	**5**	**3**
		(OR)			
15(b)		Prove that D_{70}, the set of all positive divisors of 70, is a Boolean algebra and find all its sub algebras.	**14**	**5**	**3**

PART- C (1x 10=10Marks

(Q.No.16 is compulsory)

		Marks	CO	RBT LEVEL
16	Prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.	**10**	**1**	**3**