

Reg. No.

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**B.E / B.TECH. DEGREE EXAMINATIONS, MAY 2024**

First Semester

**MA18151 – ENGINEERING MATHEMATICS – I***(Common to all branches except MR)***(Regulation 2018 / 2018A)****TIME: 3 HOURS****MAX. MARKS: 100**

- CO1** To develop the use of matrix algebra techniques which is needed for practical applications.
- CO2** To apply the skill to solve statistical problems under correlation and regression and acquire the knowledge for fitting the straight line and parabola.
- CO3** To apply the concepts of differential calculus to find the evolutes and envelopes of curves.
- CO4** To explore functions of several variables and to find the maxima and minima of functions of two or more variables.
- CO5** To acquaint the student with mathematical tools needed in evaluating multiple integrals and their usage.

**PART- A (10 x 2 = 20 Marks)**

(Answer all Questions)

- |    |  | CO | RBT<br>LEVEL |
|----|--|----|--------------|
| 1. | Find the sum and product of the Eigens value of the matrix $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ . | 1  | 2            |
| 2. | Find the matrix of the quadratic form $2x_1^2 + 5x_2^2 - 4x_1x_2$ .  | 1  | 2            |
| 3. | Given two regression lines $2y + x = 4$ and $x + 3y = 5$ find $\sigma_x$ if $\sigma_y = 2$ .                                       | 2  | 2            |
| 4. | Write the normal equations to fit a straight line $y = ax + b$ by the method of least squares.                                     | 2  | 1            |
| 5. | Find the curvature of the circle $2x^2 + 2y^2 + 5x - 2y + 1 = 0$ at any point on it.   | 3  | 2            |
| 6. | Find the envelope of the family of curves $\frac{x \cos \alpha}{a} + \frac{y \sin \alpha}{b} = 1$ where $\alpha$ is the parameter. | 3  | 2            |
| 7. | Evaluate $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} \frac{x-y}{x+y}$ .   | 4  | 2            |
| 8. | If $u = \frac{y^2}{x}$ and $v = \frac{x^2}{y}$ then find $\frac{\partial(x, y)}{\partial(u, v)}$ .                                 | 4  | 2            |
| 9. | Evaluate $\int_1^{2\sqrt{3}} \int_x^{x\sqrt{3}} xy dy dx$ .  | 5  | 3            |

10. Sketch the region of integration in  $\int_0^a \int_0^{\sqrt{a^2-y^2}} f(x,y) dx dy$ .

**PART- B (5 x 14 = 70 Marks)**

11. (a) (i) Find the Eigen values and Eigen vectors of the matrix  $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ . (7) 1 3

- (ii) Use Cayley-Hamilton theorem, to find the value of the matrix given by  $A^6 - 5A^5 + 8A^4 - 2A^3 - 9A^2 + 35A + 6I$  where  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ . (7) 1 3

**(OR)**

- (b) Reduce the quadratic form  $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$  to the canonical form through an orthogonal transformation and hence show that it is positive semi-definite. (14) 1 3

12. (a) (i) Ten students got the following percentage of marks in Physics and chemistry: (7) 2 3

Students	1	2	3	4	5	6	7	8	9	10
Marks in Physics	78	36	98	25	7 5	82	90	62	65	39
Marks in chemistry	84	51	91	60	6 8	62	86	58	63	47

Calculate the rank correlation coefficient.

- (ii) Fit a parabola, by the method of least squares, to the following data; also estimate  $y$  at  $x=6$ . (7) 2 3

$x$	1	2	3	4	5
$y$	5	12	26	60	97

**(OR)**

- (b) Obtain the equations of the lines of regression from the following data: (14) 2 3

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

13. (a) (i) Find the radius of curvature of the cycloid  $x=a(\theta+\sin\theta)$ ,  $y=a(1-\cos\theta)$ . (7) 3 3

(ii) Find the envelope of  $\frac{x}{a^2}+\frac{y}{b^2}=1$  where  $a$  and  $b$  are the parameters and are connected by the relation  $a^2+b^2=1$ . (7) 3 3

(OR)

(b) (i) Find the centre of curvature of  $y=3x^3+2x^2-3at(0,-3)$ . (7) 3 3

(ii) Find the equation of evolutes of the parabola  $x^2=4ay$ . (7) 3 3

14. (a) (i) If  $y = f(x + at) + g(x - at)$ , show that  $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$  where 'a' is constant. (7) 4 3

(ii) Expand  $f(x, y) = \sin(xy)$  in powers of  $(x - 1)$  and  $(y - \pi/2)$  through quadratic form. (7) 4 3

(OR)

(b) (i) Prove  $u = x + y + z$ ,  $v = xy + yz + zx$ ,  $w = x^2 + y^2 + z^2$  are functionally dependent and find the functional relationship between them. (7) 4 3

(ii) Using Lagrange's multiplier method, determine the dimensions of a rectangular tank, open at the top, if the surface area is 108 sq.cm. (7) 4 3

15. (a) (i) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} r^2 dr d\theta$  over the region bounded by the semicircle  $r = a \cos\theta$  above the initial line. (7) 5 3

(ii) Change of order of integration in  $\int_0^a \int_y^a \frac{x}{x^2+y^2} dx dy$  and hence evaluate. (7) 5 3

(OR)

(b) (i) Convert  $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2) dy dx$  into polar coordinates and then evaluate. (7) 5 3

(ii) Evaluate  $\int_V (x+y+z) dx dy dz$  where  $V$  is the volume bounded by the parallelepiped  $x=0, x=a, y=0, y=b, z=0, z=c$ . (7) 5 3

**PART- C (1 x 10 = 10 Marks)**

(Q.No.16 is compulsory)

Marks	CO	RBT LEVEL
(10)	4	3

16. Examine  $f(x,y) = x^3 + y^3 - 12x - 3y + 20$  for its extrême values.

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