Registration No. | | | | | | | | |

M.E./M.Tech. Degree Examinations, January 2017

First Semester

COMPUTER AIDED DESIGN

MA16183 – ADVANCED NUMERICAL METHODS

(Common to Internal Combustion Engineering)

(Regulation 2016)

QP Code:262522

Time: Three hours **Maximum : 100 marks**

Answer **ALL** questions

PART A - (10 X 2 = 20 Marks)

1. Solve the system of equation by Gauss elimination method $2x + y = 3; 7x - 3y = 4$.
2. Obtain an iterative formula to find the reciprocal of a number N using Newton's method.
3. Write the algorithm to solve first order ODE by second order Runge-Kutta method.
4. Calculate the stiffness ratio of $\frac{dy}{dx} = -100\, y, \frac{dz}{dx} = 2y - z, y(0) = 0, z(0) = 1$.
5. State the finite difference form of $2y'' + y = 5$.
6. State the Diritchlet boundary condition.
7. Classify the given partial differential equation $(x+1)u_{xx} - 2(x+2)u_{xy} + (x+3)u_{yy} = 0$
8. Write Liebmann iterative formula.
9. Give Laplace equation in polar form.
10. Define conforming element.

PART B - (5 X16 = 80 Marks)

11. (a) (i) Solve the system of linear equation using Gauss seidel method **(8)**

$$8x_1 - 3x_2 + 2x_3 = 20;\ 4x_1 + 11x_2 - x_3 = 33;\ 6x_1 + 3x_2 + 12x_3 = 35$$

(ii) Solve the following tridiagonal system $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ by **(8)**

Thomas Algorithm

(OR)

(b) Use Faddeev's method to find the eigen values of the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find its inverse. **(16)**

12. (a) Solve the initial value problem $y' = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$ at $x = 0.2, 0.4$ .Use Runge Kutta method of 4[th] order. **(16)**

(OR)

(b) (i) Using shooting technique solve the boundary value problem $y'' = y$, $y(0) = 0, y(1) = 1.17$. Compare the solution with exact solution. **(8)**

(ii) Using collocation method, approximate the solution of the equation $y'' + y + x = 0$ subject to the boundary conditions $y(0) = 0 = y(1)$ **(8)**

13. (a) (i) Solve $u_{tt} = 4u_{xx}$ with the boundary conditions $u(0,t) = 0 = u(4,t) = u_t(x,0)$ and $u(x,0) = x(4-x)$ compute u for 4 time steps with $h = 1$. **(8)**

(ii) Solve $\boldsymbol{u}_t = \boldsymbol{u}_{xx}$ with $\boldsymbol{u}(\boldsymbol{x},0) = \sin(\pi \boldsymbol{x})$ and $\boldsymbol{u}(0,\boldsymbol{t}) = \boldsymbol{u}(1,\boldsymbol{t}) = 0, \forall \boldsymbol{t} > 0$ using Crank-Nicolson method with $\boldsymbol{h} = \frac{1}{3}, \boldsymbol{k} = \frac{1}{36}$ find the solution for one time step. **(8)**

(OR)

(b) Solve the initial boundary value problem $\boldsymbol{u}_t = \boldsymbol{u}_{xx} + \boldsymbol{u}_{yy}$, $\boldsymbol{u}(\boldsymbol{x},\boldsymbol{y},0) = \cos\frac{\pi \boldsymbol{x}}{2}\cos\frac{\pi \boldsymbol{y}}{2}, -1 \le \boldsymbol{x}, \boldsymbol{y} \le 1, \boldsymbol{t} = 0$ using ADI method with $u = 0,\ x = \pm 1,\ y = \pm 1,\ t > 0,\ h = \frac{1}{2}, k = \frac{1}{6}$. **(16)**

14. (a) Solve $u_{xx} + u_{yy} = 0$ in $0 \le x \le 4, 0 \le y \le 4$ given that $u(0,y) = 0, u(4,y) = 12 + y,\ u(x,0) = 3x, u(x,4) = x^2$.Take $h = k = 1$ and **(16)**

obtain the result correct to three decimal places.

(OR)

(b) (i) Solve $\nabla^2 \boldsymbol{u} = 8\boldsymbol{x}^2 \boldsymbol{y}^2$ for the square mesh given $\boldsymbol{u} = 0$ on the 4 boundaries dividing the square into 16 sub squares of length 1 unit. **(10)**

(ii) Derive the finite difference scheme to solve $u_{xx} + u_{yy} = f(x, y)$ **(6)**

15. (a) Find one parameter Galerkin solution of the boundary value problem $\nabla^2 u = -1$ on $|x| \le 1, |y| \le 1$, $u = \frac{\partial u}{\partial n}$ on $|x| = 1, |y| = 1$ where n is the inward normal to the boundary. **(16)**

(OR)

(b) Find a two parameter Least square approximation solution of the BVP $\nabla^2 u = x^2 - 1,$ for $|x| \le 1, |y| \le \frac{1}{2}$, $u = 0$ on the boundary. **(16)**